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Reversible and Irreversible Transformations in Black-Hole Physics*

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(Received 17 September 1970)

The concepts of irreducible mass and of reversible and irreversible transformations in black holes are introduced, leading to the formula $E^2 = m_{ir}^2 + (L^2/4m_{ir}^2) + p^2$ for a black hole of linear momentum p and angular momentum L.

This note reports five conclusions: (1) The mass energy of a black hole of angular momentum L can be expressed in the form

$$m^2 = m_{ir}^2 + L^2/4m_{ir}^2, \tag{1}$$

where m_{ir} is the irreducible mass [geometrical units: $L(\text{cm}) = (G/c^3)L_{\text{conv}}(\text{gcm}^2/\text{sec})$; $m(\text{cm}) = (G/c^2)M_{\text{conv}}(\text{g})$; $G/c^2 = 0.742 \times 10^{-28}$ cm/g] of the black hole. (2) Insofar as one looks apart from the atomicity of matter one can approach arbitrarily closely to reversible transformations that augment or deplete the rotational contribution to the square of the mass. (3) The attainable range of reversible transformation extends[1,2] from $L = 0$, $m^2 = m_{ir}^2$ to $L = m^2$, $m^2 = 2m_{ir}^2$. (Contrast to the formula for mass energy as it depends upon translation, $E^2 = m^2 + p^2$, where p is unlimited; and with the formula for the squared mass energy of a meson!) (4) An irreversible transformation is characterized (Fig. 1) by an increase in the irreducible mass of the black hole. (5) There exists no process which will decrease the irreducible mass.

Roger Penrose has pointed out[3] a way to extract energy from a black hole endowed with angular momentum. It makes use of the "ergosphere" (Ruffini and Wheeler; cf. Fig. 2, reproduced from their paper[4]), the region between the horizon (surface of black hole; boundary of region from which no particle or radiation can ever escape) and the surface of infinite red shift (coincident with the horizon only for case of the angular-momentum-free Schwarzschild black hole). A particle of energy E_0 is sent from infinity into the ergosphere and decays there into (1) a particle which emerges to infinity with a rest-plus-kinetic energy E_2 greater than E_0, together with (2) a particle ("rocket ejecta") which has an energy E_1, that is negative as measured at infinity ($E_1 = E_0 - E_2$), but positive in the local Lorentz frame, and which is ejected into such a direction that it is captured into the black hole, thereby diminishing its mass. We consider the case where all masses can be regarded as infinitesimal compared with the mass of a black hole.

The energy E, as measured at infinity, of a particle of angular momentum p_φ and rest mass μ, having a turning point at r, is given by the

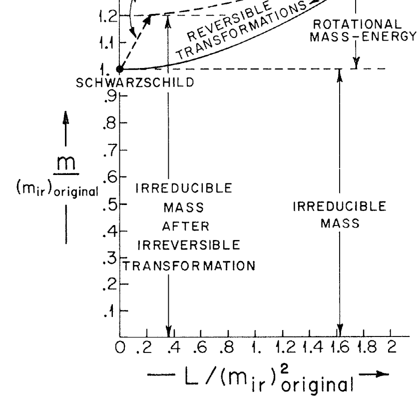

FIG. 1. Mass energy m versus angular momentum L for a black hole of specified irreducible mass m_{ir} illustrating the difference between reversible transformations and irreversible transformations (which increase the irreducible mass).

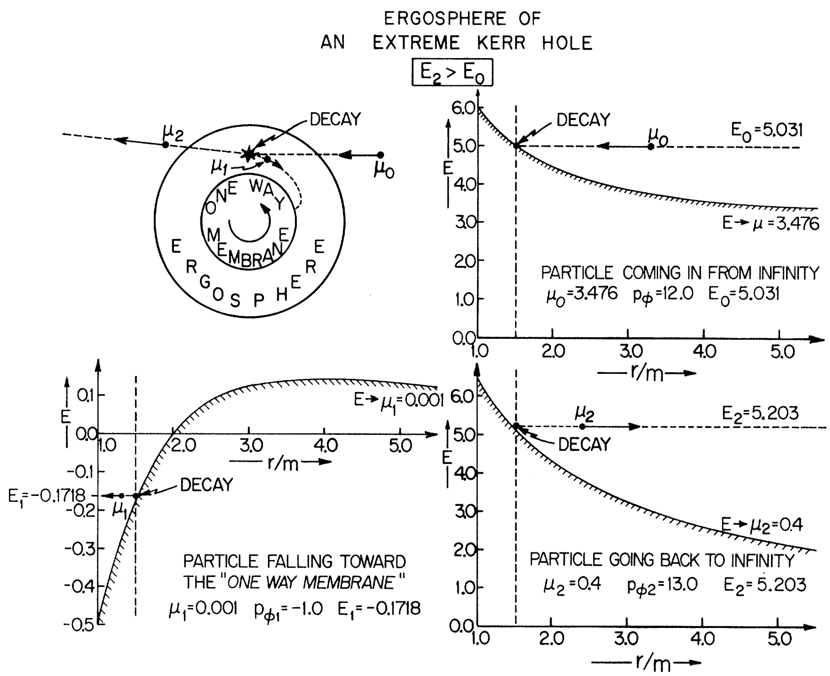

FIG. 2. (Reproduced from Ruffini and Wheeler, Ref. 4, with their kind permission.) Decay of a particle of rest-plus-kinetic energy E_0 into a particle which is captured into the black hole with positive energy as judged locally, but negative energy E_1 as judged from infinity, together with a particle of rest-plus-kinetic energy $E_2 > E_0$ which escapes to infinity. The cross-hatched curves give the effective potential (gravitational plus centrifugal) defined by the solution E of Eq. (2) for constant values of p_φ and μ.

equation[5] (where a is an abbreviation for L/m)

$$E^2[r^3+a^2(r+2m)]-4mEap_\varphi+(2m-r)p_\varphi^{\,2}-\mu^2r^2(r-2m)-a^2\mu^2r=\text{multiple of (radial momentum)}^2=0. \quad (2)$$

The Penrose process is most efficient when the reduction of mass is greatest for a given reduction in angular momentum. To meet this requirement the energy E_1 must be as negative as possible. This happens at the surface of the black hole itself,

$$r=r_+=m+(m^2-a^2)^{1/2}, \quad (3)$$

where the separation of "positive-" and negative-energy states goes to zero [vanishing of discriminant of Eq. (2) for E]. At this point the relation between energy and angular momentum reduces to

$$E_1=[a/(r_+^{\,2}+a^2)](p_\varphi)_1. \quad (4)$$

Applying the laws of conservation of energy and angular momentum to the assimilation of particle 1 by the black hole, we arrive at the relation

$$dm=\frac{(L/m)\,dL}{[m+(m^2-L^2/m^2)^{1/2}]^2+L^2/m^2}. \quad (5)$$

Integration leads to the relation

$$(1-a^2/m_2)^{1/2}=(2m_{ir}^{\,2}/m^2)^{-1}$$

which, if condition (3) is fulfilled, is equivalent to expression (1).

I would like to thank Professor J. A. Wheeler and Dr. R. Ruffini for very helpful discussions and suggestions.

*Publication assisted by National Science Foundation and U. S. Air Force Office of Scientific Research Grants to Princeton University.

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